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SOME TWO SAMPLE TESTS BASED ON ORDERED OBSERVATIONS FROM THE EXPONENTIAL DISTRIBUTION

BY

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SOME TWO SAMPLE TESTS BASED ON ORDERED OBSERVATIONS FROM

THE EXPONENTIAL DISTRIBUTION

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1. Introduction

Let $x_{11} \leq x_{12} \leq \ldots \leq x_{1n_1}$, and $x_{21} \leq x_{22} \leq \ldots \leq x_{2n_2}$, be two random samples (S_{n_1} and S_{n_2}) from populations having p.d.f.'s $f(x; A_1, \theta_1)$ and $f(x; A_2, \theta_2)$ respectively, where

$$(1) \qquad f(x;A,\theta) = \frac{1}{\theta} \exp\left[-(x-A)/\theta\right].$$

Let S_{r_1} and S_{r_2} be the sets of the first r_1 and r_2 smallest observations of S_{n_1} and S_{n_2} respectively. Then the p.d.f.'s of S_{r_1} and S_{r_2} are given, say, by $g(x_{11},\ldots,x_{1r_1}; A_1,\theta_1)$ and $g(x_{21},\ldots,x_{2r_2}; A_2,\theta_2)$, where

$$(2) \qquad g(x_1,x_2,\ldots,x_r; A,\theta) = \frac{n!}{(n-r)!} \frac{1}{\theta^r} \exp\left\{-\frac{1}{\theta}\left[\sum_{i=1}^{r}(x_i-A) + (n-r)(x_r-A)\right]\right\}.$$

The likelihood ratio tests based on the complete sets, S_{n_1} and S_{n_2} are special cases of those obtained by Sukhatme [2,3]. It can be shown that similar likelihood ratio tests based on S_{r_1} and S_{r_2} may be obtained by following Sukhatme's procedure [2]. In this report these likelihood ratio tests are reduced to equivalent tests which are expressed in terms of the well-known Chi-square and Snedecor's F distributions. Furthermore, some of the tests obtained in this report can be extended to k-sample tests.

Since percentage points for the χ^2 and F distributions are tabled, tests involving these random variables are useful in applications. We remark that the likelihood ratio test for the hypothesis H_5 (see Section 3) has been obtained by Paulson [1].

2. Preliminary lemmas.

We give several lemmas which were used to obtain the distributions of the reduced statistics. Lemmas 1-3 can be proved by the use of characteristic functions and their proofs are omitted. Proofs of lemmas 4-8 are given.

In lemmas 1, 2 and 3 below, we let $x_1 \leq x_2 \leq \ldots \leq x_r \leq \ldots \leq x_n$ be a random sample from a population having p.d.f. (1) and we define statistics u, v, and h as,

$$(3) \qquad u = \frac{2}{\theta}\left[\sum_{i=1}^{r}(x_i - A) + (n - r)(x_r - A)\right]$$

$$(4) \qquad v = \frac{2}{\theta}\left[\sum_{i=1}^{r}(x_i - x_1) + (n - r)(x_r - x_1)\right].$$

$$(5) \qquad h = \frac{2n}{\theta}(x_1 - A).$$

Lemma 1. u is distributed as $\chi^2(2r)$.

Lemma 2. v is distributed as $\chi^2(2r-2)$.

Lemma 3. v and h are independently distributed as $\chi^2(2r-2)$ and $\chi^2(2)$ respectively.

Lemmas 4-8 deal with the case of two samples. The statistics u_1, v_1 and u_2, v_2 are defined as in (3) and (4). Three additional variables w_1, w_2, and w are defined in (6), (7) and (8).

$$(6) \qquad w_1 = \frac{2n_1}{\theta_1}(x_{11} - x_{21}), \quad \text{for} \quad x_{11} > x_{21}$$

(7) $$w_2 = \frac{2n_2}{\theta_2}(x_{21} - x_{11}), \quad \text{for} \quad x_{21} > x_{11}$$

(8) $$w = w_1, \text{ when } x_{11} > x_{21} \text{ and } w = w_2, \text{ when } x_{21} > x_{11}.$$

Lemma 4. If $A_1 = A_2$, then

(9) $$\Pr(x_{11} > x_{21}) = \frac{n_2/\theta_2}{n_1/\theta_1 + n_2/\theta_2}$$

and

(10) $$\Pr(x_{21} > x_{11}) = \frac{n_1/\theta_1}{n_1/\theta_1 + n_2/\theta_2}$$

Proof:

$$\Pr(x_{11} > x_{21}) = \int_{A_1}^{\infty}\int_{A_1}^{x_{11}} \frac{n_1 n_2}{\theta_1 \theta_2} e^{-\frac{n_1}{\theta_1}(x_{11}-A_1) - \frac{n_2}{\theta_2}(x_{21}-A_1)} dx_{11} dx_{21}$$

$$= \frac{n_2/\theta_2}{n_1/\theta_1 + n_2/\theta_2} .$$

Hence,

$$\Pr(x_{21} > x_{11}) = 1 - \Pr(x_{11} > x_{21}) = \frac{n_1/\theta_1}{n_1/\theta_1 + n_2/\theta_2}$$

Lemma 5. If $A_1 = A_2$, then both w_1 (given that $x_{11} > x_{21}$) and w_2 (given that $x_{21} > x_{11}$) are distributed as $\chi^2(2)$.

Proof. Since $A_1 = A_2$, w_1 can be written as

$$w_1 = \frac{2n_1}{\theta_1}\left[(x_{11} - A_1) - (x_{21} - A_2)\right].$$

Consequently,

$$x_{11} - A_1 = \frac{\theta_1}{2n_1} w_1 + (x_{21} - A_2).$$

Let $x_{11} - A_1 = y_1$ and $x_{21} - A_2 = y_2$, then the condition that $x_{11} > x_{21}$ is equivalent to $y_1 > y_2$. Since the joint distribution of y_1 and y_2 is, say

$$f(y_1, y_2) = \frac{n_1 \cdot n_2}{\theta_1 \cdot \theta_2} e^{-\frac{n_1}{\theta_1} y_1 - \frac{n_2}{\theta_2} y_2}, \quad y_1, y_2 > 0, \tag{11}$$

we have

$$Pr(w_1 \leq w \mid y_1 > y_2) = \frac{Pr(w_1 \leq w, \ y_1 > y_2)}{Pr(y_1 > y_2)}. \tag{12}$$

According to lemma 4

$$Pr(y_1 > y_2) = Pr(x_{11} > x_{21}) = \frac{n_2/\theta_2}{n_1/\theta_1 + n_2/\theta_2}.$$

Further, it is readily verified that

$$Pr(w_1 \leq w, \ y_1 > y_2) = \frac{n_2/\theta_2}{n_1/\theta_1 + n_2/\theta_2}\left[1 - e^{-w/2}\right]. \tag{13}$$

Therefore,

$$Pr(w_1 \leq w \mid y_1 > y_2) = 1 - e^{-w/2} \tag{14}$$

But, by (17)

$$\Pr(w_1 \leq w_{10}, x_{11} > x_{21}) = \Pr(x_{11} > x_{21}) \left[1 - e^{-\frac{1}{2} w_{10}}\right]$$

and by lemma 2, $\Pr(V_1 \leq V_{10})$ and $\Pr(V_2 \leq V_{20})$ are cumulative χ^2-distributions with $(2r_1 - 2)$ and $(2r_2 - 2)$ d.f.'s. Thus lemma 7 is proved.

Lemma 8. If $A_1 = A_2$ then V_1, V_2 and w are independently distributed as $\chi^2(2r_1 - 2)$, $\chi^2(2r_2 - 2)$ and $\chi^2(2)$ respectively.

Proof. Since

$$\begin{aligned}\Pr(V_1 \leq V_{10}, V_2 \leq V_{20}, w \leq w_o) &= \Pr(V_1 \leq V_{10}, V_2 \leq V_{20}, w \leq w_o, x_{11} > x_{21}) \\ &\quad + \Pr(V_1 \leq V_{10}, V_2 \leq V_{20}, w \leq w_o, x_{11} < x_{21})\end{aligned}$$

then by (17) lemma 8 follows.

3. Likelihood ratio tests and equivalent reduced tests.

The various hypotheses and their associated likelihood ratio and equivalent reduced tests are listed below. The details involved in obtaining the likelihood ratio will not be given here, since they are well known.

A. Statement of hypotheses:

a) H_1: To test $\theta_1 = \theta_2$
(assuming A_1 and A_2 are known).

b) H_2: To test $\theta_1 = \theta_2$
(assuming $A_1 = A_2$).

c) H_3: To test $\theta_1 = \theta_2$.

d) H_4: To test $A_1 = A_2$
(assuming θ_1 and θ_2 are known).

e) H_5: To test $A_1 = A_2$ (assuming $\theta_1 = \theta_2$).

f) H_6: To test $A_1 = A_2$.

g) H_7: To test $A_1 = A_2$ and $\theta_1 = \theta_2$.

B. Likelihood ratio tests

In a), b) and c) below we let

$$K = \prod_{i=1}^{2} \left(\frac{r_1 + r_2}{r_i} \right)^{r_i} \tag{19}$$

a) For H_1:

$$\lambda_1 = K \left[(1 + c_1)^{r_1} \left(1 + \frac{1}{c_1}\right)^{r_2} \right]^{-1} \tag{20}$$

where

$$c_1 = \frac{\sum_{j=1}^{r_2} (x_{2j} - A_2) + (n_2 - r_2)(x_{2r_2} - A_2)}{\sum_{j=1}^{r_1} (x_{1j} - A_1) + (n_1 - r_1)(x_{1r_1} - A_1)} \tag{21}$$

b) For H_2:

$$\lambda_2 = K \left[(1 + c_2)^{r_1} \left(1 + \frac{1}{c_2}\right)^{r_2} \right]^{-1}, \quad \text{if } x_{11} < x_{21}$$

$$(22) \qquad = K \left[\left(1 + \frac{1}{c_2'}\right)^{r_1} (1 + c_2')^{r_2} \right]^{-1}, \quad \text{if } x_{21} < x_{11}$$

where

e) For H_5:

(28)
$$\lambda_5 = (1 + c_5)^{-(r_1+r_2)} \quad \text{if} \quad x_{11} > x_{21}$$
$$= (1 + c_5')^{-(r_1+r_2)} \quad \text{if} \quad x_{11} < x_{21}$$

where

(29)
$$c_5 = \frac{n_1(x_{11} - x_{21})}{\sum_{i=1}^{2} \left[\sum_{j=1}^{r_i} (x_{ij} - x_{i1}) + (n_i - r_i)(x_{ir_i} - x_{i1}) \right]}$$

$$c_5' = \frac{n_2(x_{21} - x_{11})}{\sum_{i=1}^{2} \left[\sum_{j=1}^{r_i} (x_{ij} - x_{i1}) + (n_i - r_i)(x_{ir_i} - x_{i1}) \right]}$$

f) For H_6:

(30)
$$\lambda_6 = (1 + c_6)^{-r_1} \quad \text{if} \quad x_{11} > x_{21}$$
$$= (1 + c_6')^{-r_2} \quad \text{if} \quad x_{11} < x_{21}$$

where

$$c_6 = \frac{n_1(x_{11} - x_{21})}{\sum_{j=1}^{r_1} (x_{1j} - x_{11}) + (n_1 - r_1)(x_{1r_1} - x_{11})}$$

(31)

$$c_6' = \frac{n_2(x_{21} - x_{11})}{\sum_{j=1}^{r_2} (x_{2j} - x_{21}) + (n_2 - r_2)(x_{2r_2} - x_{21})}.$$

g) For H_7:

$$\lambda_7 = \prod_{i=1}^{2} \left(\frac{\hat{\theta}_i}{\hat{\theta}} \right)^{r_i}$$

where

$$\hat{\theta}_i = \frac{1}{r_i} \sum_{j=1}^{r_i} (x_{ij} - x_{i1}) + (n_i - r_i)(x_{ir_i} - x_{i1})$$

$$\hat{\theta} = \frac{1}{r_1 + r_2} \sum_{i=1}^{2} \left[\sum_{j=1}^{r_i} (x_{ij} - \hat{A}) + (n_i - r_i)(x_{ir_i} - \hat{A}) \right]$$

and where $\hat{A} = \min (x_{11}, x_{21})$.

C. Reduced Tests

By the use of the lemmas in section 2, $\lambda_1, \lambda_2, \ldots, \lambda_6$ can be reduced to the following equivalent tests having the corresponding distributions (see Table 1). The authors have not succeeded in reducing λ_7 to a known test.

TABLE 1

Hypothesis	Equivalent reduced Tests	Distributions	Critical regions
H_1	$f_1 = \frac{r_1}{r_2}\ c_1$	$F(2r_2, 2r_1)$	(2)
H_2	$f_2 = \frac{r_1-1}{r_2}\ c_2$, if $x_{11} < x_{21}$	$F(2r_2, 2r_1-2)$	(2)
	$f_2' = \frac{r_2-1}{r_1}\ c_2'$, if $x_{21} < x_{11}$	$F(2r_1, 2r_2-2)$	(2)
H_3	$f_3 = \frac{r_1-1}{r_2-1}\ c_3$	$F(2r_2-2, 2r_1-2)$	(2)
H_4	$f_4 = c_4$	$\chi^2(2)$	(1)
H_5	$f_5 = \frac{2r_1+2r_2-4}{2}\ c_5$, if $x_{11} > x_{21}$	$F(2, 2r_1+2r_2-4)$	(1)
	$f_5' = \frac{2r_1+2r_2-4}{2}\ c_5'$, if $x_{21} > x_{11}$	$F(2, 2r_1+2r_2-4)$	(1)
H_6	$f_6 = \frac{2r_1-2}{2}\ c_6$, if $x_{11} > x_{21}$	$F(2, 2r_1-2)$	(1)
	$f_6' = \frac{2r_2-2}{2}\ c_6'$, if $x_{21} > x_{11}$	$F(2, 2r_2-2)$	(1)

In Table 1 numbers in the "critical regions" column indicate that the reduced tests obtained may be either one-sided or two-sided. For example, consider the case where $r_1 = r_2 = 10$ and $\alpha = .05$. Then for the various H_i, $i = 1,2,3,4,5,6$ we have the following critical regions which are summarized for convenience in Table 2.

TABLE 2

Critical Regions

H_1:	$f_1 > 2.46$ or $f_1 < \frac{1}{2.46}$
H_2:	$f_2 > 2.56$ or $f_2 < \frac{1}{2.50}$ when $x_{11} < x_{21}$
	or $f_2' > 2.56$ or $f_2' < \frac{1}{2.50}$ when $x_{21} < x_{11}$
H_3:	$f_3 > 2.60$ or $f_3 < \frac{1}{2.60}$
H_4:	$f_4 > 5.99$
H_5:	$f_5 > 3.26$ when $x_{11} > x_{21}$
	$f_5' > 3.26$ when $x_{21} > x_{11}$
H_6:	$f_6 > 3.55$ when $x_{11} > x_{21}$
	$f_6' > 3.55$ when $x_{21} > x_{11}$

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